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Conjugate Effect of Convection and Conduction in a Nanofluid–Filled Complicated Cavity

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Abstract

Convective and conductive flow and thermal behaviors of nanofluid inside a complicated square enclosure is investigated numerically. The cavity consists of a centered heated diamond shaped solid obstacle. The vertical walls are uniformly heated at constant temperature while the base surface of the cavity is insulated. The upper horizontal part of the enclosure is surrounded by solid material with constant thermal conductivity. The upper boundary of this solid region is considered as adiabatic. The working fluid is water based nanofluid having alumina nanoparticle. The integral forms of the governing equations are solved numerically using Galerkin's Weighted Residual Finite Element method. Computational domains are divided into finite numbers of body fitted control volumes with collocated variable arrangement. Results are presented in the form of average Nusselt number, average temperature, mean velocity of the nanofluid, mid height horizontal and vertical velocities for a selected range of convective parameter Rayleigh number (10^3 - 10^6). Streamlines and isothermal lines are also displayed for the above mentioned parameter. The results indicate that the highest heat transfer rate is found for the greatest Rayleigh number.

Keywords: *Free convection, conduction, complicated enclosure, nanofluid, finite element method.*

1. Introduction

The fluids with solid-sized nanoparticles suspended in fluids are called “nanofluids”. Due to small sizes and very large specific surface areas of the nanoparticles, nanofluids have superior properties like high thermal conductivity, minimal clogging in flow passages, long-term stability, and homogeneity. Thus, nanofluids have a wide range of potential applications like electronics, automotive, and nuclear applications where improved heat transfer or efficient heat dissipation is required.

House et al. [1] considered natural convection in a vertical square cavity with heat conducting body, placed on center in order to understand the effect of the heat conducting body on the heat transfer process in the cavity. They found that the heat transfer across the enclosure enhanced by a body with thermal conductivity ratio less than unity. Nasrin [2] analyzed numerically flow and heat transfer characteristics for wavy enclosure with a centered heat conducting cylinder. The author showed that the considered parameters strongly affect the flow phenomenon and temperature field inside the chamber. Conducting largest obstacle was preferable for effective heat transfer mechanism. Conjugate natural convection in a square porous cavity was studied by Baytas et al. [3]. A numerical study of convection heat transfer for air from two vertically separated horizontal heated cylinders confined to a rectangular enclosure had been conducted by Lacroix and Joyeux [4], where the vertical walls have finite conductivity and the heat conduction has been taken into consideration. Dong and Li [5] studied conjugate of natural convection and conduction in a complicated enclosure. They concluded that the flow and heat transfer increased with the increase of the thermal conductivity in solid region as well as both geometric shape and Rayleigh number affected the overall flow and heat transfer greatly.

Saleh et al. [6] investigated effect of conduction in bottom wall on Darcy–Bénard convection in a porous enclosure where increasing either the Rayleigh number or the thermal conductivity ratio or both, and decreasing the thickness of the bounded wall could increase the average Nusselt number for the porous enclosure. Conjugate natural convection around a finned pipe in a square enclosure with internal heat generation was studied by Ben-Nakhi and Chamkha [7]. Result illustrated that the effects of the finned pipe inclination angle and fins length on the streamlines and temperature contours within the enclosure were significant. Kuznetsov and Sheremet [8] analyzed conjugate natural convection in an enclosure with local heat sources where the governing unsteady, three-dimensional heat transfer equations, written in dimensionless terms of the vorticity vector, vector potential functions and temperature, had been solved using an implicit finite-difference method. Conduction with natural convection heat transfer across multi-layer building blocks was studied by Baig and Antar [9]. The effect of both techniques could compensate for removing the insulation layer in the cavity used to increase the wall thermal resistance. Parvin et al. [10] analyzed heat transfer by nanofluid with different geometries. From their study it was observed that nanofluid enhanced rate of heat transfer than base fluid (clear water). Conjugate natural convection in an inclined nanofluid ‑ filled enclosure was performed by Aminossadati and Ghasemi [11]. The result showed that the utilization of the nanofluid enhanced the thermal performance of the enclosure and that the length of the centered block affected the heat transfer rate.

The present study introduces heat transfer phenomena by water based nanofluid having Al_2O_3 nanoparticles. The numerical computation covers a wide range of Rayleigh number ($10^3 \leq Ra \leq 10^6$).

2. Model Specification

As shown in Fig. 1, a horizontal high temperature (T_h) diamond shaped hollow cylinder of diameter d is enclosed by an enclosure of square cross-section. Both vertical side walls in Fig. 1 of this cross-section are isothermal, the temperature is T_c. The upper horizontal boundary part of the rectangular is surrounded by solid material with constant thermal conductivity k_s. The upper of this solid region horizontal boundary is considered as adiabatic. The lower horizontal boundary of the cavity is supposed as ideal adiabatic boundary with infinite small wall thickness and zero thermal conductivity. It is assumed that the nanoparticles are spherical shaped and diameters are less than 10nm. All dimensions and boundaries of this geometrical model are noted in Fig. 1. The operational fluid is water based nanofluid having alumina nanoparticles. The thermophysical properties of the nanofluid are taken from Ogut [12] and given in Table 1.

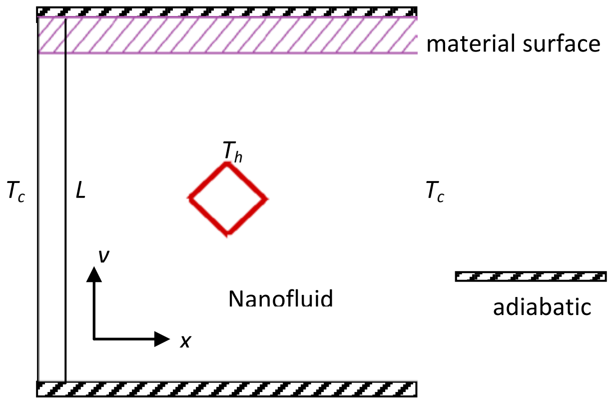

Fig. 1. Physical model of the complicated cavity

Table 1: Thermo physical properties of base fluid and alumina nanoparticles

Physical Properties	Fluid phase (Water)	Al_2O_3
C_p(J/kgK)	4179	765
ρ (kg/m^3)	997.1	3970
k (W/mK)	0.613	40
$\alpha\times10^7$ (m^2/s)	1.47	131.7
$\beta\times10^5$ (1/K)	21	2.4

3. Mathematical Formulation

A two-dimensional, steady, laminar, incompressible, free convection flow is considered within the enclosure and the fluid properties are assumed to be constant. The gravitational force acts in the vertically downward direction. The nanofluid containing two nanoparticles is assumed incompressible and the flow is considered to be laminar. It is taken that water and both nanoparticles are in thermal equilibrium and no slip occurs between them. The density of the nanofluid is approximated by the Boussinesq model. Only steady state case is considered. The radiation effects are taken as negligible. The dimensional equations describing the flow under Boussinesq approximation are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\rho_{nf}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu_{nf}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$\rho_{nf}\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu_{nf}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+g\left(\rho\beta\right)_{nf}(T-T_c) \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

For solid material region the energy equation is

$$\frac{k_m}{\left(\rho C_p\right)_{nf}}\left(\frac{\partial^2 T_m}{\partial x^2}+\frac{\partial^2 T_m}{\partial y^2}\right)=0 \tag{5}$$

Here the effective density $\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s$ (6)

the effective heat capacitance $\left(\rho C_p\right)_{nf}=(1-\phi)\left(\rho C_p\right)_f+\phi\left(\rho C_p\right)_s$ (7)

the effective thermal expansion coefficient $(\rho\beta)_{nf}=(1-\phi)(\rho\beta)_f+\phi(\rho\beta)_s$ (8)

the effective thermal diffusivity $\alpha_{nf}=k_{nf}\big/\left(\rho C_p\right)_{nf}$ (9)

the effective dynamic viscosity (modified form) of Brinkman model [12]

$$\mu_{nf}=\mu_f\,(1-\phi)^{-2.5} \tag{10}$$

and the effective thermal conductivity (modified form) of Maxwell Garnett (MG) model [14]

$$k_{nf}=k_f\,\frac{k_s+2k_f-2\phi\left(k_f-k_s\right)}{k_s+2k_f+\phi\left(k_f-k_s\right)} \tag{11}$$

The boundary conditions for the present problem can be written as follows:
at all solid boundaries: $u = 0$, $v = 0$
at the bottom of the cavity: $\frac{\partial T}{\partial y}=0$
at the top of the material surface: $\frac{\partial T_m}{\partial y}=0$
at the vertical walls: $T = T_c$
at the diamond shaped boundaries: $T = T_h$
at the fluid-solid interface: $\left(\frac{\partial T}{\partial y}\right)_{nanofluid}=\frac{k_m}{k_{nf}}\left(\frac{\partial T_m}{\partial y}\right)_{material}$

The above equations are non-dimensionalized by using the following dimensionless quantities

$$X=\frac{x}{L},\ Y=\frac{y}{L},\ D=\frac{d}{L},\ U=\frac{uL}{\nu_f},\ V=\frac{vL}{\nu_f},\ P=\frac{pL^2}{\rho_f\,\nu_f^2},\ \theta=\frac{(T-T_c)}{(T_h-T_c)},\ \theta_m=\frac{(T_m-T_c)}{(T_h-T_c)}$$

Then the governing equations take the non-dimensional form given below

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{12}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\rho_f}{\rho_{nf}}\frac{\partial P}{\partial Y}+\frac{\nu_{nf}}{\nu_f}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right) \tag{13}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\rho_f}{\rho_{nf}}\frac{\partial P}{\partial Y}+\frac{\nu_{nf}}{\nu_f}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+\frac{Ra}{Pr}\frac{(1-\phi)(\rho\beta)_f+\phi(\rho\beta)_s}{\rho_{nf}\,\beta_f}\theta \tag{14}$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{Pr}\frac{\alpha_{nf}}{\alpha_f}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{15}$$

For heat generating obstacle the energy equation is

$$\frac{K}{Pr}\left(\frac{\partial^2\theta_m}{\partial X^2}+\frac{\partial^2\theta_m}{\partial Y^2}\right)=0 \tag{16}$$

where $Pr=\frac{\nu_f}{\alpha_f}$ is the Prandtl number, $Ra=\frac{g\beta_f L^3(T_h-T_c)}{\nu_f\,\alpha_f}$ is the Rayleigh number and $K=\frac{k_m}{k_{nf}}$ is the dimensionless ratio of the thermal conductivities of solid and nanofluid.

The boundary conditions for the present problem are specified as follows:

at all solid boundaries: $U = 0$, $V = 0$

at the bottom of the cavity: $\frac{\partial\theta}{\partial Y}=0$

at the top of the material surface: $\frac{\partial\theta_m}{\partial Y}=0$

at the vertical walls: $\theta = 0$

at the diamond shaped boundaries: $\theta = 1$

at the fluid-solid interface: $\left(\frac{\partial\theta}{\partial Y}\right)_{nanofluid}=K\left(\frac{\partial\theta_m}{\partial Y}\right)_{material}$

The normal temperature gradient can be written as $\frac{\partial\theta}{\partial N}=\sqrt{\left(\frac{\partial\theta}{\partial X}\right)^2+\left(\frac{\partial\theta}{\partial Y}\right)^2}$

The average Nusselt number at the diamond shaped heated boundaries may be expressed as $Nu=\frac{1}{S}\int_0^S \overline{Nu}dS$

where N, S and dS are the non-dimensional distances either X or Y direction acting normal to the surface, arc length and coordinate along the cylinder surface respectively.

4. Computational Procedure

The penalty finite element method [15] is used to solve the Eqs. (2) - (4), where the pressure P is eliminated by a penalty constraint. The continuity equation is automatically fulfilled for large values of this penalty constraint. Then the velocity components (U, V), temperature (θ) and concentration (C) are expanded using a basis set. The Galerkin finite element technique yields the subsequent nonlinear residual equations. Three points Gaussian quadrature is used to evaluate the integrals in these equations. The non-linear residual equations are solved using Newton–Raphson method to determine the coefficients of the expansions. The convergence of solutions is assumed when the relative error for each variable between consecutive iterations is recorded below the convergence criterion ε such that $\left|\psi^{n+1}-\psi^{n}\right|\le 10^{-4}$, where n is the number of iteration and Ψ is a function of U, V, θ and C.

5. Results and Discussion

The natural convection and conduction phenomenon inside a complicated cavity filled with water alumina nanofluid is influenced by different Rayleigh number Ra. Analysis of the results is made through obtained streamlines, isotherms, average Nusselt number, mean temperature and velocity of the nanofluid, mid height horizontal and vertical velocities of nanofluid for various Ra. The ranges are varied as $10^3 \le Ra \le 10^6$, while the other parameters Pr, K and ϕ, are kept fixed at 6.2, 1 and 5% respectively.

Thermal current and flow activities are offered in Fig. 2 (a)-(b) with different Raleigh number (Ra). The strength of the flow circulation and the thermal current activities is much more activated with increasing Ra. For higher Ra, the isothermal lines take an onion shape from the diamond shaped body to the solid material surface. This is due to releasing the thermal boundary layer for the effect of higher buoyancy force. The lines become more concentrated near the heated obstacle which indicates steep temperature gradients and hence, an increase in the overall heat transfers from the heated surface to the surroundings of the complicated cavity. On the other hand, two main circulating vortices cover the whole region of the cavity without the upper material surface at $Ra = 10^3$. The right and left vortices near the diamond shaped cylinder rotate in clockwise and counterclockwise direction respectively. Size of spinning cells in the velocity field becomes larger and they become more strengthen due to buoyancy force for increasing Ra from 10^3 to 10^6. In addition more perturbation is observed in the streamlines at $Ra = 10^6$ because of rising buoyancy force.

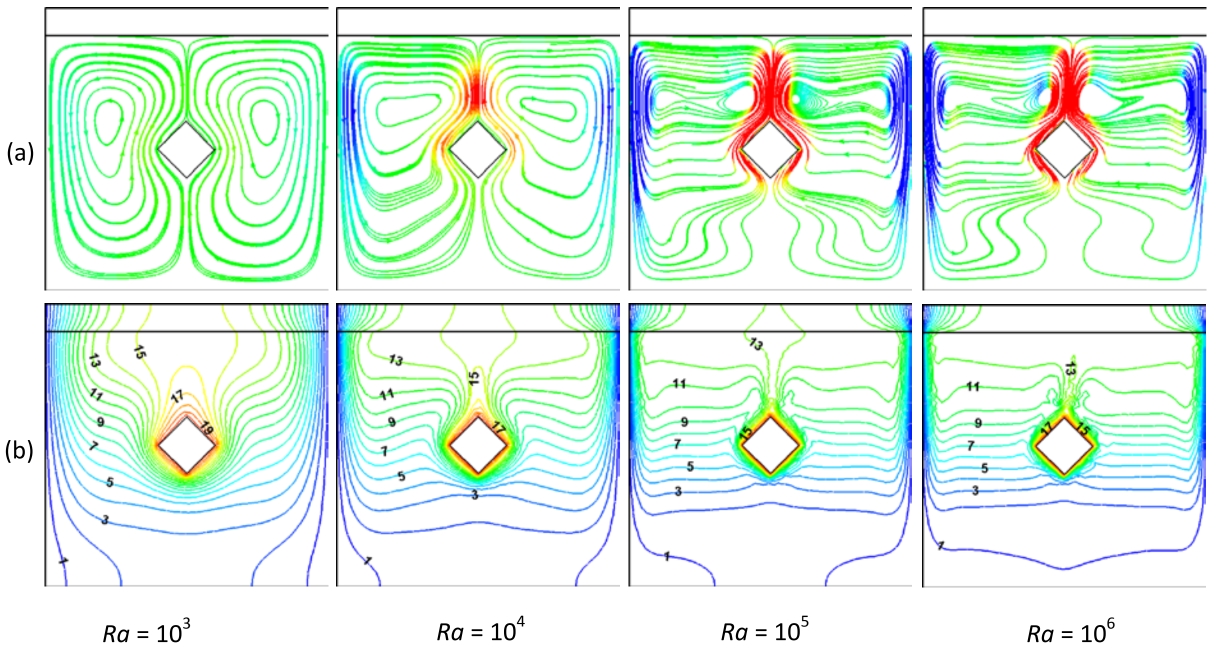

Fig. 2. Effect of Raleigh number on (a) Isotherms and (b) Streamlines

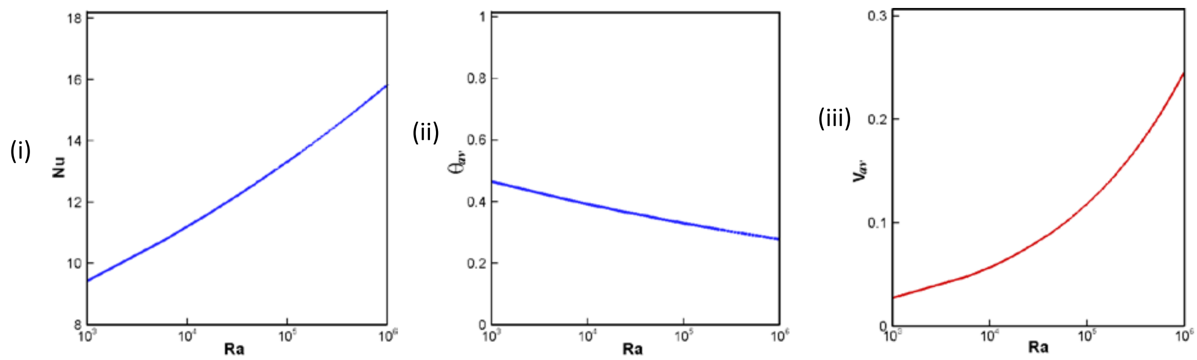

Fig. 3. (i) Average Nusselt number (ii) mean temperature and (iii) mean velocity of nanofluid for various *Ra*

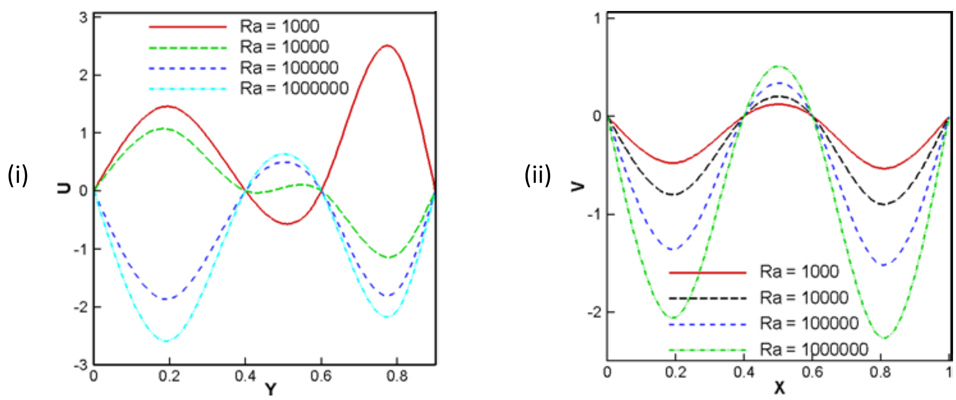

Fig. 4. Plot of mid height (i) *U* velocity and (ii) *V* velocity for various *Ra*

The variation of the average Nusselt number (*Nu*) at the centered heated cylinder, mean temperature of the nanofluid (θ_{av}) and average velocity of the nanofluid (V_{av}) for different Rayleigh numbers have been presented in Fig. 3(i)-(iii). It is clearly seen from the figure that the average Nusselt number is highest for the largest $Ra = 10^6$. This is because, the fluid with the highest Rayleigh number is capable to carry more heat away from the heated surface and dissipated through the whole domain. On the other hand, the rate of heat transfer for water based nanofluid having alumina nanoparticles is found to be more effective than the base fluid (clear water) due to higher thermal conductivity of solid nanoparticles. For this reason, rate of convective heat transfer enhances by 38% for the effect of *Ra*.

The mid height horizontal (U) velocity at X = 0.5 and the vertical (V) velocity at the middle (Y = 0.05) of the complicated geometry for various Ra is displayed in Fig. 4 (i)-(ii). Significant variation in velocity is found due to changing the Ra. It is observed that for the lowest Ra (= 10^3) the horizontal velocity line graphs are less varied than others. The waviness in the V-X profile devalues for lower values of Ra.

6. Conclusion

The following conclusions may be drawn from the present investigation:

The effect of buoyancy parameter Ra on isotherms and streamlines are remarkable.

Escalating Ra enhances the average Nusselt number at the heated diamond shaped cylinder.

The mean temperature of the nanofluid devalues for rising Ra.

Higher average velocity of the working fluid is found for higher Ra.

7. Acknowledgement

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8. References

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